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The Power of Algebraic Thinking

Whenever we visit the zoo or see animals in the wild, we are witnesses to amazing phenomena. Many animals are physically superior to humans. Some are stronger, others are faster, and several have far more acute senses. Nevertheless, human beings continue to dominate the world.

Although we are not as strong as many animals, we are able to invent and construct important tools that compensate for our lack of strength. We are slower than many animals, but we have created wheels and energy-powered engines that allow us to outpace any animal. We can even make tools that allow us to see things our normal senses are incapable of revealing. It is our mental power that makes all of these inventions a reality.

Just as we are able to create physical tools to compensate for our natural shortcomings, we also have access to tremendously powerful mental tools. In order to meet any challenge, even to solve mathematical problems, we first must acquire information through our sensory organs--mainly our eyes, and next through our ears. Then the information is transferred and stored in our working memory for further processing. To create and improve those tools that help us find solutions to numerical problems, and we need to further investigate both the strengths and the weaknesses of our vision and memory systems.

Our ability to perceive and discriminate among objects within a visual field is limited to only five or six items at a time. The polygons shown here have only three or four sides. It is not a problem to recognize the total number of sides at a glance. However, once the number of the sides exceeds five or six, a quick assessment becomes more difficult. We have to laboriously examine the sides one by one in order to count them. Do you know why we always use hyphens or parentheses to separate our phone numbers into groups of three or four digits? It is because they're easier to read that way. We often would like to rewrite the expression 7 + 7 + 7 + 7 + 7 as the shortcut, 5 x 7, meaning five sevens. This is because 5 x 7 is much easier to read, write, perceive, and assess than the original expression in long addition.

Also, there is what is referred to as the "memory overload problem." Have you ever forgotten someone's name immediately after being introduced? What happens if you are interrupted while counting something? Most people will have to start all over again because they can't remember the place in the count that they were at when they stopped just a moment earlier. How many of us need to create a shopping list just before leaving home to go to the store? From these everyday examples, you can see just how limited our short-term memory actually can be.

To get a better understanding of these limits, let's look at a simple arithmetic problem before investigating why algebra is such an indispensable tool to deal with complex numerical problems.
Most people would agree that the following are two simple math routines for mathematicians:

$$67 + 32 =$$
$$67 \times 32 =$$

The addition problem is so simple that most of us arrive at the answer, 99, almost immediately. However, the multiplication problem appears to be much more difficult. Few of us can do this calculation in our heads. The truth is that most mathematicians cannot do it in their heads either. How can a routine

multiplication problem cause trouble for even highly respected mathematicians? The answer may surprise you. It has nothing to do with knowledge of math. Because it takes so many more steps to calculate 67 x 32 than 67 + 32, we soon run out of working memory.

Most real-life problems contain many more quantities, unknowns, and relationships than the simple multiplication problem, 67 x 32. Thus it seems that we need more help dealing with these issues than ever.

How do we deal with the perception and memory overload problems in our daily life? Everywhere we look, we see signs. They were designed as symbols to represent thoughts that ask for an action in response. Once we learn what each symbol represents, it will always deliver that message, and require the same response every time it catches our eye. It is natural to use a shorter message--or find a symbol, to replace a lengthy verbal description whenever possible.

For example, when we see a bright red, octagon-shaped sign with the letters S-T-O-P written in white, we immediately think "slow down" and prepare to stop our bike, skateboard, or car. The stop-sign symbol replaces the need for a lengthy sequence of instructions and perceptions. Our everyday use of signs and symbols provides a hint of the answer to why we use a single letter, called a "variable," to represent an unknown quantity in an equation. A single letter is the shortest readily available symbol.

Once we shorten the name for an unknown quantity, the next task is for us to translate a long verbal description of relationships into a concise symbolic statement, which we call an "equation." By now, we have shortened the lengthy verbal expression to its minimal form. By writing the equation on a piece of paper or another media, we are easing our memory-overload problem.

Every day, we are presented with many problems. For the simplest ones, we figure out the solution directly from the verbal description. For those that involve many quantities and relationships, we have to rely on tools such as the ones described above; this in turn is the foundations of what we call algebra. Once we change the linguistic expressions into equations, our remaining task is to figure out how to find the related solutions using those equations. To find a solution means to find the value of the variable that makes both sides of the equation equal.

You may not be able to appreciate the full potential of algebra when using it to solve simple problems. It seems to be more trouble than it's worth. Yet we use simplified problems in the beginning of the learning process, so that students can practice and master the use of this valuable tool. You may not see the advantage of driving an automobile if all you need to do is travel back and forth between your house and your neighbor's house. In fact, it is often easier and faster to just walk over.

However, to travel from San Francisco to New York City, there is no doubt that a car is much more useful than our feet, and an airplane will get you there even faster. Basic arithmetic is only sufficient to solve simple, small-scale problems. When the problems become more complex and grow into large-scale projects, we need algebra to help us solve them. By introducing a variable, then making an equation out of the verbal language, and finally solving for the variable, we have a procedure that turns the solving of complex problems into a series of symbolic steps that can be easily understood and utilized.

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